Dynamic scaling for avalanches in disordered systems

Guang-Ping Zheng and Mo Li
Department of Materials Science and Engineering, The Johns Hopkins University, 3400 North Charles Street, Baltimore, Maryland 21218
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Dynamic scaling for fracture or breakdown process in disordered systems is investigated in a two-dimensional random field Ising model (RFIM). We find two evolving stages in the avalanche process in the RFIM. At the short-time regime, a power-law growth of the avalanche size Δs is observed; and at late times, the conventional nucleation and growth process is found. At the critical point of the RFIM, the avalanche size is found to obey the dynamic scaling law $\Delta s \sim t^{(d-\beta/\nu)/z}$. From this dynamic scaling relation, the critical strength of the random field D_c and the critical exponents, β, ν, and z, are determined. The observed dynamics is explained by a simple nucleation theory of first-order phase transformations.

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I. INTRODUCTION

Study of fracture or breakdown of disordered or heterogeneous systems under external perturbation is of great interest due to its direct relevance to many practical problems [1–3]. It covers a wide range of fields from deformation and fracture in metallic glasses [4], Barkhausen jump in ferromagnets [5], martensite transformation in shape-memory alloys [6], earthquakes [7], and deformation of granular materials [8] to the behavior of index or exchange rates in the stock market [9]. The fracture or breakdown in these heterogeneous systems occurs when the external control parameter approaches its threshold and is usually preceded by precursors in the form of avalanches [10]. The underlying corporative behavior in the avalanches, such as the self-organized criticality phenomenon, has been investigated extensively in some of these systems [3,11]. However, most of the works to date have concentrated on the equilibrium or static scaling behavior [10–13,16–19]; very limited attention has been paid to how an avalanche develops. As a result, the dynamic nature of the avalanche or fracture remains unsettled.

Fracture or breakdown is, by nature, a dynamic process. The time-dependent behavior of the avalanches bears a significant bearing on our understanding of deformation and fracture that depends sensitively on the external loading history, as well as the intrinsic disorder embedded in the system. One of such cases that we are interested in, but far less studied, is deformation and fracture in metallic glasses [13]. Deformation in metallic glasses occurs when the applied stress exceeds the yield or flow stress. Due to the lack of well-defined flow defects, such as dislocations, deformation in metallic glasses proceeds by the formation of localized zones with heavy concentration of plastic shearing inside [13]. Different-sized shear zones occur during the deformation process and eventually evolve into a large, spanning shear band traversing the sample, resulting in fracture of the material. Much like the Barkhausen jump in ferromagnets [5], the formation and propagation of the local shear zones lead to the serrated flow manifested in the stress-strain curve. Each rise and fall in the curve corresponds to the propagation of a shear zone, or an avalanche, which of course depends sensitivity on the resolution of the instruments. The exact origin of the formation of the shear band is still not well understood. Certain structural and chemical heterogeneities present in metallic glasses are suspected to be the cause of shear banding [4,13]. The phenomenology of the deformation process suggests that it is a collective behavior of the local shear zones that lead to the shear banding or fracture: either it is a result of coalescence and growth of many local deformation avalanches or an abnormal runaway propagation of one of the deformed zones that leads to the percolating phenomenon. One of the fundamental questions underpinning the deformation and fracture mechanisms is, therefore, how these deformation zones initiate in the early stages, and what their dynamic behaviors are in later times.

The dynamic properties, such as initiation and propagation of the local events or avalanches, whether it is a local magnetic domain [5] or a local shear deformation zone [13], are, therefore, of great importance to both theoretical understanding and practical applications [4–13]. Fracture and deformation process have been treated as phase transitions where the free energy difference between deformed or fractured and undeformed or perfect systems constitutes the driving force for the transition [14–16]. The scaling laws were shown in a mean-field model that treat the fracture or breakdown point as a spinodal point in first-order transition [14]. However, depending on the specific models used, the scaling is different for different model systems.

Another unsettled issue about the analogy made between the phase transitions and fracture is that one of the salient features of the fracture or breakdown processes, the *instability*, may be overlooked. Certainly as the system is approaching the breakdown point or fracture, it could be well described by the analogous models of phase transitions. But beyond the point of the yielding or breakdown, the system might be driven by the runaway process, or instability, characterized by *irreversibility*. At and past this point, the characteristic time for the thermal fluctuations is shorter than that of the fracture process. Therefore, thermodynamic description of the fracture or breakdown process in the form of phase transitions may not be adequate, which includes both the thermally activated and disorder-induced fracture.

This dilemma motivates us to look for other model systems that possess the two essential characteristics for a fracture or breakdown process: *local interactions* (not necessarily the free energy) and *instability*. The interaction in the random field Ising model is the spin-spin coupling constant J; the heterogeneity or disorder is introduced by the random field h_i. The fracture or breakdown process is described in the random field Ising model (RFIM) by the formation of avalanches: when the external applied field increases, the spins in the regions with strong disorder will flip first. The flipped spins will further trigger their neighbors to flip as the local environments of the (unflipped) spins are changed due to the presence of the flipped spins. This (almost) instabilitylike process is termed *avalanche*.

As is known, the RFIM indeed captures some of the essence of the fracture process [19], but its dynamic properties remain poorly understood. In addition, the scaling exponents are not easy to obtain, especially for those in two dimensions (2D) [19]. Furthermore, the distinction between the early-stage nucleation of the avalanches and the propagation or growth at later times has not been clearly made and used in the equilibrium scaling analysis, although a spinodal instability is predicted for the fracture or breakdown point [14]. This work will therefore focus primarily on the dynamic behavior of the avalanche process in the RFIM at zero temperature. Moreover, we expect to obtain, using the dynamic scaling, the equilibrium scaling exponents that are very difficult to obtain directly from static scaling.

This paper is organized as follows. In the next section, we introduce the RFIM model used in this work and the procedure for dynamic scaling. In Sec. III, we describe the detailed algorithms employed in this work and present the major results obtained from the simulation. In Sec. IV, we discuss the results and present a simple theoretical explanation for the two-stage evolution in the dynamic process observed in our work. Finally, we conclude this work by a brief summary.

II. DYNAMIC SCALING FOR AVALANCHE IN THE MODEL SYSTEM

A. The model

In this work, we use a two-dimensional random-field Ising model to describe the development of avalanches in disordered systems under an external field. The Hamiltonian of this system can be written as

$$\hat{H} = -J\sum_{\langle ij\rangle} S_iS_j - \sum_i h_iS_i - H\sum_i S_i, \tag{1}$$

where $S_i = \pm 1$ are spin variables and $\langle ij\rangle$ denotes the summation extending over all nearest-neighbor spins. J is the spin-spin coupling constant. H is a homogeneous external applied magnetic field, h_i is an uncorrelated random field that represents the internal disorder and is chosen from a Gaussian distribution: $\langle h_i\rangle = 0$ and $\langle h(x)h(x')\rangle = 2D\delta(x-x')$. Both H and D are in units of $J(>0)$.

Since the avalanche process is in general much faster than thermal fluctuation, we focus on the system at $T=0$. It has been found in this case that at $H=0$, the system exhibits a continuous phase transition at D_c below which there is a ferromagnetic order state [17]. When a sweeping external field is applied [18,19], there exists an infinite avalanche in the hysteresis loop if $D<D_c$; at $D>D_c$ only finite-size avalanches are found. At the critical random-field strength $D=D_c$, there are avalanches of all sizes that satisfy statistical distributions in the form of power laws.

D_c and corresponding critical exponents have been determined at $H=0$ by finding the exact ground states using a maximum-flow, minimum-cut algorithm in graph theory [17] or by the equilibrium power-law scaling under a sweeping field [18,19]. However, the values of D_c in these studies are found to lie in a wide range from 0.54 to 0.75, and the critical exponents are still not well obtained. It needs to be pointed out that the aforementioned results in two-dimensional RFIM are all obtained from the equilibrium calculation and simulation. To our knowledge, there has been no dynamic study of avalanches in the RFIM so far.

In this work we report that under an external field, the nonequilibrium relaxation of metastable states shows a dynamic scaling behavior at early times. From these results, we are able to determined D_c and the critical exponents using dynamic scaling for the avalanche process in 2D. In addition, our results show that this method is more efficient than the equilibrium techniques in obtaining the critical exponents.

B. Dynamic scaling

Our prediction of dynamic scaling is based on the finite-size dynamic scaling hypothesis [20]. Near the critical point (D_c, H_c) in the phase diagram [21], the order parameter that is defined to be the magnetization jump $m(t)\equiv[M(t)-M_c(D_c)]/2$ satisfies the finite-size scaling relation [19–21],

$$m(L,t) \sim L^{-\beta/\nu}F(L/\xi(t)), \tag{2}$$

where $M(t)\equiv\langle(\Sigma S_i)/L^d\rangle$ is the total magnetization, $\langle\,\rangle$ denotes average over the random-field configurations, M_c is the value at critical point, and β/ν measures the dimension of m. L is the lattice size and $\xi(t)$ is the nonequilibrium spatial correlation length of the flipped spins at time t. F is the scaling function. During the avalanche process in a finite-size system, we can define the starting time $t=0$ when the largest avalanche starts and the total number of flipped spins equals $s(L,t)$ during time regime (0, t]. At the beginning of evolution, $\xi(t)$ is small compared with L. Therefore, the avalanche size $s(L,t)\sim L^d m(L,t)$. Together with Eq. (2) we have

$$\begin{aligned} s(L,t) &\sim L^d m \\ &\sim L^{d-\beta/\nu}F[(H_c-H)L^{\phi/\nu},(D-D_c)L^{1/\nu};tL^{-z}], \end{aligned} \tag{3}$$

where ϕ/ν measures the dimension of H. z is the dynamic critical exponent and is defined by the dynamic scaling hypothesis $\tau=\xi^z f(k\xi)$, where τ is the relaxation time of metastable state, and $k\sim 2\pi/L$ is the wave vector of the system. Near the critical point, this relation gives us the finite-size scaling for the duration time of avalanche [22].

$$\langle t_0\rangle \sim L^z. \tag{4a}$$

At the critical point, we have from Eq. (3)

$$\langle s(t)\rangle \sim t^{[d-\beta/\nu]z} \sim t^{\theta}. \tag{4b}$$

This scaling relation is valid at least at the short-time regime. If we take the logarithm on both sides of Eq. (3), the derivative with respect to $r \equiv (D_c - D)/D_c$ gives

$$\left.\frac{\partial \ln\langle s(t)\rangle}{\partial r}\right|_{r=0} \sim t^{1/\nu z} \tag{4c}$$

at the critical point $r=0$ and $H=H_c$.

Scaling relation (4b) shows that at (D_c, H_c), the avalanche size increases with a power law in time. Equations (4) enable us to find the critical value of D_c and critical exponents β, ν, and z by measurements of the dynamic behavior of avalanches along the critical line $\{(D,H_c)\}$ [21].

III. SHORT-TIME DYNAMIC SCALING FOR AVALANCHES

A. Algorithms

We use synchronous dynamics to investigate the kinetics of avalanche where all spins $\{S_i\}$ are updated simultaneously. First the local field $\{f_j\}$ is calculated, $f_i = \Sigma S_j + h_i + H$; then the following rules of renewal are applied to each spin: (i) If $f_i = 0$, the spin-flip probability of S_i is 0.5; (ii) otherwise, $S_i = \mathrm{sgn}(f_i)$.

One time step in the simulation is defined as one attempt of all the spin updates. The initial configuration of the system is a ferromagnetic state at $H=0$ with all spins up $(M=1)$. The system is then allowed to evolve under a varying external field described by the following procedures. (i) The time is set to zero $(t=0)$, then the external field is decreased by ΔH and is kept constant; (ii) at time t, synchronous dynamics is applied and the avalanche size $s(t)$ is calculated using the relation $s(t) = L^2[M(0) - M(t)]/2$; (iii) Step (ii) is repeated until time t_0 when the metastable state is achieved, i.e., none of the spins flip at time step t_0 (t_0 is defined as the duration time).

Procedures (i), (ii), and (iii) are carried out until the magnetization saturates at $M=-1$. In a finite-size system the critical field $H_c(L)$ is defined as the value at which the total avalanche $s(t_0)$ is the largest or the duration time t_0 is the longest. Therefore, the avalanche evolution is recognized as $s(t)$ at $H_c(L)$.

In this work, we used two methods to change the field. The first is to fix ΔH at a constant value throughout the field-sweeping process. The external field is varied as a step function and the driving rate can be measured by ΔH. The second approach is to adjust ΔH to let it be the local field of the most unstable spin. This corresponds to infinitely slow driving or quasistatic driving. In this case, only one spin is flipped at the beginning of each avalanche. Therefore, the evolution of the system is deterministic: For a configuration $\{h_i\}$, the evolution of avalanches is reproducible.

$s(t)$ is averaged over 100–5000 random-field configurations, depending on the system size. $H_c(L)$ defined from largest avalanche sizes $\langle s(t_0)\rangle$ and the longest duration time $\langle t_0\rangle$ has different values near D_c, but the dynamic scaling relations [Eqs. (4)] do not change. We give the result of $s(t)$ here that is defined as the avalanche process whose duration time t_0 is the longest during a magnetization reversal process. We also measure the avalanche size as the numbers of flipped spins. The same dynamic scaling is found near D_c.

B. Results

Figure 1(a) is the evolution of avalanches at $H_c(L)$ in the short-time regime at different values of the random-field strength under an infinitely slow driving field. In a finite-size system, the critical random-field strength $D_c(L)$ is defined such that the avalanche time $\langle t_0\rangle$ has a maximum at $D_c(L)$ [Fig. 1(c)]. It shows that at $D_c(L)$, $m(t)$ fits well to the power law in time, as predicted by Eq. (4b). This gives us an efficient way to locate $D_c(L)$ by comparing the derivation of the $m(t)$ curves with that of the power-law fits, as shown in Fig. 1(c). Figure 1(b) shows the effect of driving rate ΔH. Although θ decreases with increasing ΔH, it can be seen from Fig. 1(c) that the value of $D_c(L)$ determined is not affected by ΔH. Both $D_c(L)$ and θ are listed in Table I.

Figure 2(a) is the log-log plot of Eq. (4c) under infinitely slow driving. $\langle t_0\rangle$ is fitted to Eq. (4a), as shown in the inset. The exponents θ, ν, and z can be determined by fitting to Eqs. (4) in a finite-size system, and are extrapolated to $L \to \infty$. Table I lists the results from fitting θ and νz at $L = 512$, 1024, and 2048.

The critical random-field strength $D_c(\infty)$ in an infinite RFIM can be calculated by the finite-size scaling relation,

$$D_c(L) - D_c(\infty) \sim L^{-1/\nu}. \tag{5}$$

The result is $D_c(\infty) = 0.65 \pm 0.03$ as shown in Fig. 2(b). Once we have determined $D_c(\infty)$, we are able to confirm the finite-size scaling relation (3). Figure 3 shows at $D_c(\infty)$, the scaling of $m(t)$ between a pair of lattices. The exponents β/ν and z can be calculated and averaged to compare with those determined by Eqs. (4a)–(4c). Table I lists all the results.

The exponents and D_c determined from dynamic scaling of the avalanche agree with those calculated from ground state finding in 2D RFIM without magnetic field [17] and other methods [18,19]. It is found that finite-size scaling has significant effect on the avalanche in RFIM [19]. In this work, however, we did not find the strong finite-size effect on the dynamic scaling behavior of the avalanche process at short time. This can be easily understood by the fact that our dynamic scaling is at the short-time regime of avalanche evolution. At this stage, the avalanche is still small compared with the system size.

One of the consequences of the early stage dynamics is that the exponents calculated from Eqs. (4) have less statistical error than those in equilibrium simulation studies. This suggests to us that in low dimensions, this method may serve as a very efficient and reliable alternative to study the critical phenomena in RFIM. In Table II we compare the exponents in two-dimensional RFIM determined by different methods.

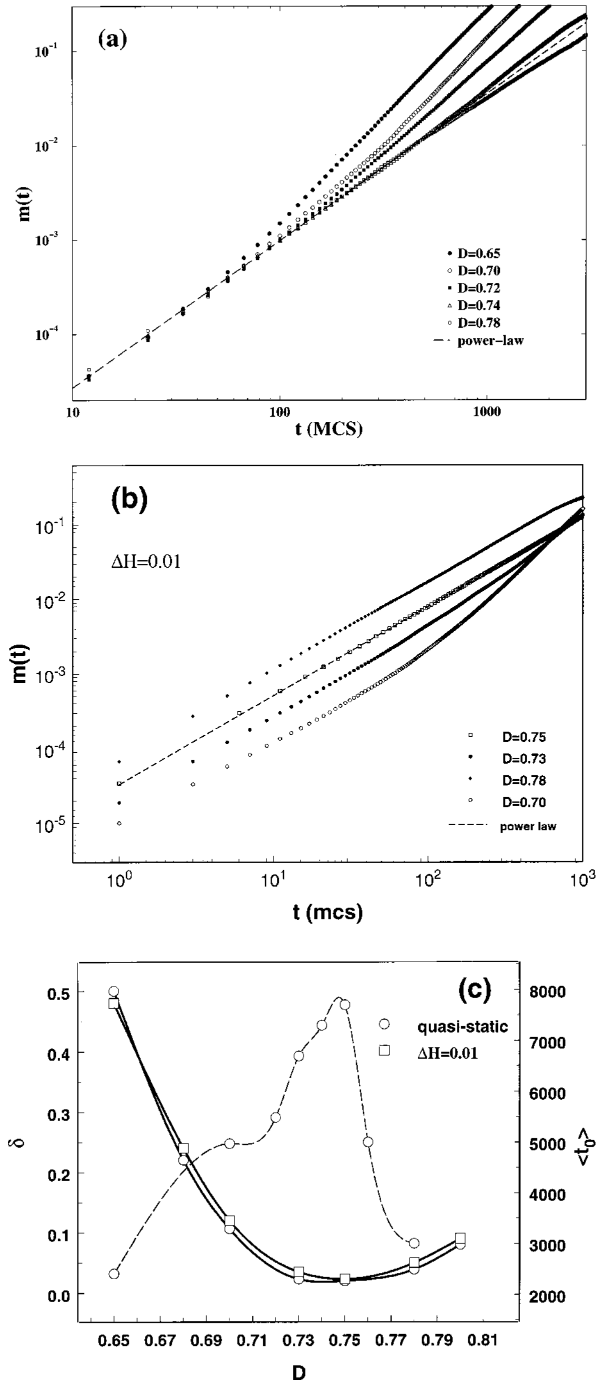

FIG. 1. The log-log plot of the evolution of avalanches at early stage for the RFIM at $L=1024$. (a) Quasistatic driving and (b) $\Delta H=0.01$. (c) is the square derivation δ from power-law fits (solid line) and the avalanche time $\langle t_0 \rangle$ (dash line). The minimum δ leads to $D_c(L)$, which is consistent with that determined by the maximum avalanche time.

TABLE I. The exponents and D_c determined from dynamic scaling relations (4) and finite-size scaling relation (3). Unless specified, the exponents are determined from critical dynamic scaling for avalanche under quasistatic driving.

L	$D_c(L)$ $(\Delta H=0.01)$	$D_c(L)$	θ $(\Delta H=0.01)$	θ	z	β/ν	$1/\nu z$
256	1.02	1.05					
512	0.870	0.870		1.60	1.30	0.13	
1024	0.750	0.755	1.17	1.56	1.27	0.12	0.72
2048	0.705	0.710	1.20	1.52	1.27	0.10	0.77

IV. DISCUSSION

To explain the short-time scaling behavior during the breakdown process in disordered systems, we first show in Fig. 4 the fitting of $m(t)$, which is equivalent to the transition fraction f in a first-order phase transformation (FOPT), to the classical Kolmogorov-Johnson-Mehl-Avrami (KJMA) equation [23]

$$f=1-\exp(bt^n), \tag{6}$$

where b is a constant that depends on the nucleation rate, $n=2$ at the beginning of FOPT if all nuclei are distributed

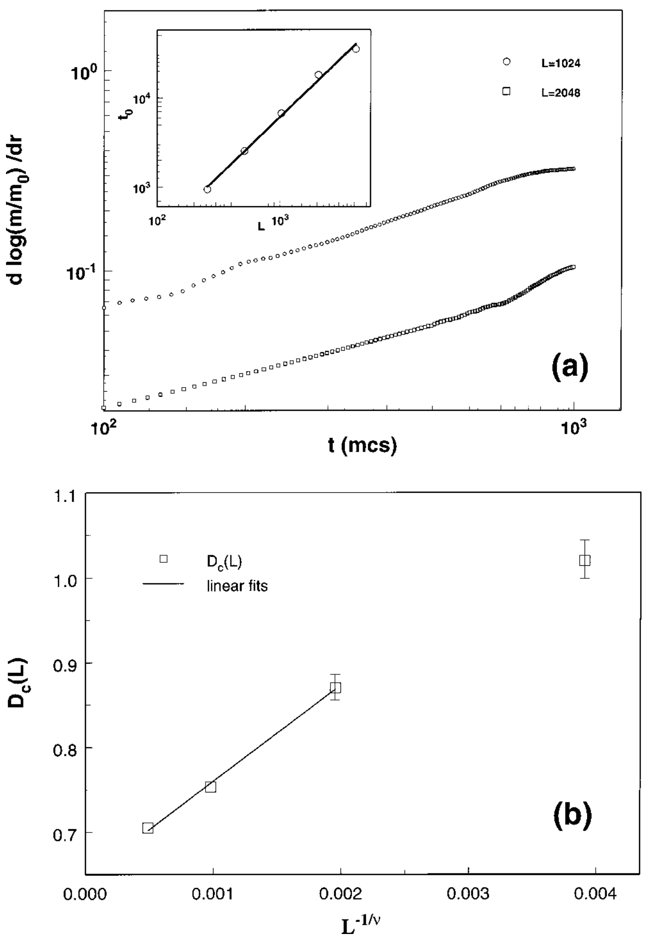

FIG. 2. (a) Determination of exponents z and νz by short-time scaling at $D_c(L)$. The dots represent the results calculated from the curve $\ln(m) \sim t$ at $D=0.73$, 0.74, and 0.75, and are extrapolated to $r=0$ with $L=1024$. The squares represent those at $D=0.69$ and 0.70 with $L=2048$. The inset shows the duration time of avalanche at $D_c(L)$. The plots are in log-log scale. (b) Relation between $D_c(L)$ and $L^{-1/\nu}$. The results of linear fits are $D_c(\infty)=0.65 \pm 0.03$ and $\nu=1.0\pm 0.1$.

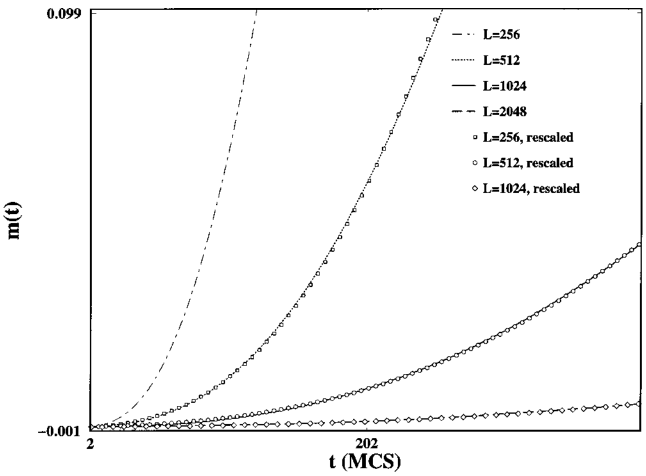

FIG. 3. Determination of exponents z and β/ν by finite size scaling at $D_c(\infty)=0.65$. The system is under quasistatic driving. The symbols are the rescaled data by factors $2^{\beta/\nu}$ to m and 2^z to t, respectively. The scaling exponents determined by a pair of lattices are listed in Table I.

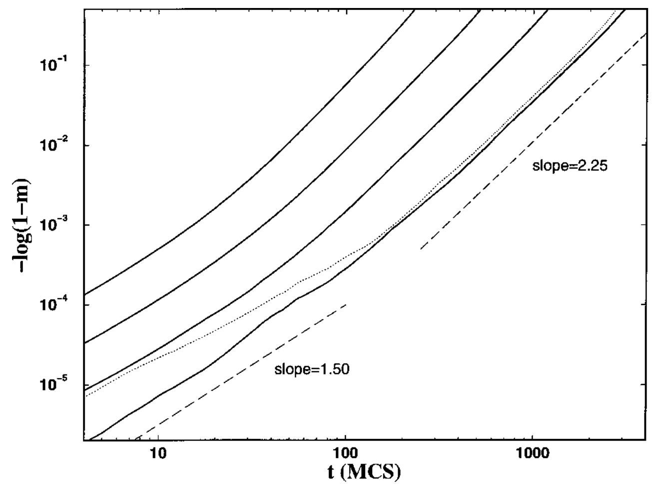

FIG. 4. The log-log plot of $-\ln(1-m)$ vs t. m is the transition fraction. Solid lines: $L=256$, 516, 1024, and 2048 (from above) under quasistatic driving at $D=D_c(\infty)$. Dotted line: $\Delta H=0.01$, $L=2048$. The dashed lines are plots of Eq. (6) with the slope equal to n.

randomly. Figure 4 plots $\ln[^{-}\ln(1-m)]$ vs $\ln(t)$ in Eq. (6). It shows clearly that there are *two stages* in the evolution of avalanches: In the short-time regime, n is much smaller than 2, while at the late stage, $n\approx 2.25\pm 0.03$, which is consistent with the KJMA theory (where n is between 2 and 3 in 2D). Therefore, the late stage breakdown process in RFIM is the growth or coalescence process of small avalanches randomly distributed over the system. The short-time evolution of the avalanches is unique for the disorder media. (Note that the KJMA equation is satisfied by the field-driven FOPT in a pure Ising model [24].) Furthermore, the driving rate of the external field seems to affect the evolution in short times while at a latetime it is irrelevant.

Our explanation for this two-stage dynamic process is that at the short-time regime, the evolution of the avalanche behaves like a diffusion process. The local random fields act as Brownian forces to the roughening surfaces of the avalanches. Note that the spreading of the avalanche is anisotropic. At the short times the avalanche size grows as $s(t)\sim w(t)\zeta$, where $w(t)$ is the width of avalanche and $w(t)\sim t^{1/2}$ for a random growth process [25]. ζ is the average transverse extent of the flipped spin domain and it may grow as $\zeta\sim t^{1/z}$. Therefore we have $s(t)\sim t^{\theta}$ at the early stage and θ is smaller than 2 in two-dimensional RFIM. Because ΔH affects the distribution of small flipped spin domains at the beginning of avalanche [26], θ is affected by the driving rate of the external field.

The existence of the early stage dynamics process in the fracture or breakdown process in the RFIM appears to support the analogy made between fracture and the spinodal nucleation for a first-order phase transition [14]. The unique dynamic scaling behavior in the short-time regime suggests that fractures or avalanches are nucleation processes controlled largely by a ''diffusionlike'' mechanism. This regime is distinct from the later stages where the confirmation of the dynamic behavior with the KJMA model unambiguously points to the growth or propagation of the avalanches.

For the short-time dynamic scaling, we focused primarily on the largest avalanche. Since this avalanche size is usually defined as an order parameter in the nonequilibrium RFIM [18,19,21], the critical exponents determined by our dynamic scaling can be compared with those obtained by the static critical scaling [18,19]. It is remarkable to see that this short-time power-law scaling $s(t)\sim t^{\theta}$ is universal for any ava-

TABLE II. $D_c(\infty)$ and critical exponents determined by different methods. A is from our dynamic scaling; B1 and B2 are from equilibrium critical scaling; C is from ground state finding.

	$D_c(\infty)$	z	β	$1/\nu$	θ or ([a]) $(d-\beta/\nu)/z$
A	0.65 ± 0.03	1.27 ± 0.03	0.10 ± 0.04	1.0 ± 0.1	1.50 ± 0.02
B1[b]	0.75 ± 0.03	1.3 ± 0.2	0.2 ± 0.2	0.63 ± 0.04	1.4 ± 0.2
B2[a]	0.54 or 0			0.13 ± 0.13	1.56 ± 0.05
C[c]	0.64 ± 0.08		-0.038 ± 0.0009	0.50 ± 0.02	

[a]Reference [19] z in 2D was not measured but $(d-\beta/\nu)/z$ was conjectured to be equal to $1/\sigma\nu z$ and $\sigma\nu z=0.64\pm 0.02$.
[b]Reference [18].
[c]Reference [17].

lanche, i.e., θ is the same for avalanches in all sizes. This issue will be further discussed elsewhere [27].

V. SUMMARY

We studied dynamic scaling for the avalanche process in RFIM using numerical simulations. We found that the dynamics has two stages. In the short-time regime, diffusive growth or nucleation of the avalanche is found; at a later stage, the dynamics is consistent with the KJMA growth mechanism. Particular attention is paid to the early stage of the breakdown process. In the short-time regime, the avalanche size is found to obey the dynamic power-law scaling $s(t)\sim t^{(d-\beta/\nu)/z}$. This power-law evolution turns into the dynamic behavior described by the KJMA equation in later times. In the thermodynamic limit, the crossover time τ from the power-law regime to the KJMA regime is expected to diverge as the strength of the random field approaches the critical value D_c. These results support the proposition that the disorder-driven system at zero temperature should behave like the homogeneous system driven by thermal perturbation close to a spinodal point.

Using the short-time dynamic scaling near the critical point, we are able to determine the critical strength of random field D_c and related critical exponents, β, ν, and z in the two-dimensional RFIM that have been found very difficult to obtain using static scaling. Our work, although numerical in nature, suggests an efficient way of obtaining the equilibrium scaling exponents for disordered systems.

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[21] K. Dahmen and J. P. Sethna, Phys. Rev. Lett. **71**, 3222 (1993); The critical point is defined by two relevant variables, $r=D_c-D$ and $h=H_c(D_c)-H$, and is the end point of the infinite avalanche lines [Fig. 1(a) in this paper]. But we should point out that this critical point is analogous to the spinodal point $[T<T_c,H_s(T)]$ in an Ising model rather than its critical point $(T_c,0)$ since $H_c(D_c)\neq 0$ in RFIM at $T=0$.

[22] The relaxation time τ is the crossover time from short-time evolution to late stage growth. But in this study, we let z to be defined by finite-size scaling for the duration time of avalanche.

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